# TWO METHODS FOR MODEL UPDATING USING DAMAGE RITZ VECTORS

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**ABSTRACT**: We propose two methods for determining the change [ΔK] in the linear stiffness properties of a structure through measurement of several of the lower modes of free vibration. The problem we consider is the following: assumed given is a baseline undamped N degree of freedom linear structural dynamics model for which the system mass matrix, stiffness matrix, natural frequencies, and mode shapes are known. It is assumed that due to aging, damage or other mechanisms, the stiffness properties are altered in a localized manner, and that the location of this property change is known. The objective is to determine the change [ΔK] in the stiffness matrix by measuring m of the lower modes at some number n > m coordinates. Two methods are proposed for this purpose.

The first method utilizes "damage Ritz vectors" or DRV's to obtain estimates for the expanded (N-dimensional) measured mode shapes. The DRV is defined in such a way that it is directly related to the localized damage. Expressing the expanded measured mode shapes of the altered system as linear combinations of the undamaged mode shapes and the DRV turns out to provide a simple yet accurate way to do mode shape expansion. The DRV mode shape expansion is combined with an iterative, residual - based scheme for estimating the stiffness change [ΔK] and for updating the expanded mode shapes. An example showing quick, accurate convergence is presented for an eight degree of freedom spring mass model.

The second updating method uses a *reduced* system model of the same dimension m as the measured modes, so that mode shape expansions are obviated. An updating procedure is proposed in which two quantities are determined iteratively: 1) the stiffness property change [ΔK] of the full, N-DOF model and 2) the slave/master coordinate transformation needed to do the model reduction for the altered system. The method converges quickly and accurately for the example case presented.

## NOMENCLATURE

| | |
|---|---|
| {b} | damage location vector (Nx1) |
| [K] | stiffness matrix, undamaged system (NxN) |
| [ΔK] | change in stiffness matrix |
| $[K_u]$ | altered stiffness matrix = [K] - [ΔK] |
| [M] | mass matrix, damaged and undamaged systems |
| $[k_{ss}]$, $[k_{sm}]$, $[k_{ms}]$, $[k_{mm}]$ | partitioned stiffness sub-matrices |
| $[\tilde{m}]$, $[\tilde{k}]$ | reduced mass, stiffness matrices (mxm) |
| {r} | damage Ritz vector |
| {R} | residual vector |
| [R] | reduction matrix |
| [T] | slave/master coordinate transformation matrix |
| {x} | model coordinate vector (Nx1) |
| $\{x\}_s$ | slave coordinate vector (sx1) |
| $\{x\}_m$ | master coordinate vector (mx1) |
| [W] | mode shape expansion matrix |
| [θ] | DRV augmented modal matrix |
| {φ} | undamaged system mode shape |
| {ψ} | damaged system mode shape |
| $\{\ \}_r$ | rth mode property |
| $\{\tilde{\ }\}$, $[\tilde{\ }]$ | reduced mode shape, modal matrix |
| {^}, [^] | expanded mode shape, modal matrix |
| $\omega_r$ | rth mode natural frequency, baseline system |
| $\omega_{er}$ | rth mode natural frequency, damaged system |
| α | scalar change in spring stiffness |
| m | number of measured modes |
| n | number of measured coordinates |
| N | degrees of freedom, full model |
| s | number of slave degrees of freedom |

## 1. INTRODUCTION

The baseline or "undamaged" structural system is assumed to be represented by the undamped linear model

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \tag{1}$$

where the NxN symmetric mass and stiffness matrices [M] and [K] are assumed known, as are the natural frequencies $\omega_r$ and mode shapes $\{\phi\}_r$, where r is the mode number. It is assumed that the m lowest frequency modes of the damaged or altered structure are reconstructed experimentally through measurement of n coordinates, with n assumed greater than m. Thus, the available data are the m measured mode shapes $\{\tilde{\psi}\}_r$, of dimension nx1, and the associated natural frequencies $\omega_{er}$. The model updating techniques often require that the measured mode shapes be expanded to the full system dimension N before these measured modes are used to do model updating. The expanded mode shapes of the damaged or altered system are denoted by $\{\hat{\psi}\}_r$. Assuming we have a reliable way to expand the measured mode shapes, the residuals for each mode are defined as

$$\{R\}_r = \left([K] - \omega^2{}_{er}[M]\right)\{\hat{\psi}\}_r = [\Delta K]\{\hat{\psi}\}_r \qquad (2)$$

The modal residual defined in equation (2) can be calculated, as all terms are known. If the measured mode shape expansion is done exactly, then the modal residual vector defined in equation (2) will have zero entries except in those slots for which the stiffness matrix is altered. Ideally the nonzero residual entries in the residual vectors will thus identify the location of the damage. If the mode shape expansion is not done exactly, however, then additional contributions to the residuals will be made by the errors in the expanded mode shapes. Thus, in general the residuals will have contributions from both the change $[\Delta K]$ in stiffness and from any errors in the expansion of the measured modes. One needs to extract that portion of the residual which is due to property change in order to calculate the change in system properties.

One direct method for estimating the change $[\Delta K]$ in the stiffness matrix is the minimum rank perturbation theory (MRPT) relation [1]

$$[\Delta K] = \{R\}_r\left(\{R\}_r^{\,T}\{\hat{\psi}\}_r\right)^{-1}\{R\}_r^{\,T} \qquad (3)$$

where the calculation will generally produce a different result $[\Delta K]$ for each measured mode. Some related updating methods are discussed in [2-5], and [6,7] and references cited therein are also relevant.

A number of methods have been proposed for doing expansion of the measured modes. See, for example, Levine-West et al [8] and Alvin [2]. Here we use static Ritz vectors as a way to do the modal expansion. Ritz vectors have also been used extensively by Cao and Zimmerman [4,9]. In their procedure the static Ritz vector is calculated as the coordinate displacement pattern resulting when a set of static loads is applied at the same location as the vibration exciters which are eventually used to excite the structure. A sequence of orthonormal dynamic Ritz vectors is then calculated, using the static Ritz vector as the generator. The result is a set of orthogonal basis vectors which can be used to represent the measured mode shapes. Cao and Zimmerman have also presented a way to determine the set of Ritz vectors from experimental data [9]. The Ritz vectors used in [4,9] are one set of basis vectors one could use to approximate the measured mode shapes. The more obvious choice for this set of trial vectors would be the undamaged system mode shapes; Cao and Zimmerman concluded, however, that the Ritz vector set provides a better basis than do the undamaged mode shapes. In our work we also utilize Ritz vectors, but we define them so that they *characterize* the damage, that is, they essentially define how the damage alters the mode shapes (Cao and Zimmerman's Ritz vectors are not related to the damage). The construction we use follows the work of Chu and Milman [10], who used Ritz vectors very effectively to characterize the effect of isolated, localized damping/stiffness elements added to a large space truss in only a few locations.

The rest of this paper contains the following sections: Section 2 describes the mode shape expansion procedure using "damage Ritz vectors" or DRV's. Section 3 describes an iterative model updating method utilizing the Ritz vector based mode shape expansion. Section 4 describes an updating procedure based on use of the reduced dynamic model of the damaged structure. Section 5 contains concluding remarks.

## 2. DAMAGE RITZ VECTOR - BASED MODE SHAPE EXPANSION

To fix ideas we consider the eight degree of freedom spring-mass system shown in Figure 1, for which all of the undamaged system masses and stiffnesses have values of unity. This example will be used throughout this paper. For reference the two lowest mode shapes and frequencies of this undamaged system are the following:

$\{\phi\}_1 = [.0891\ .1752\ .2554\ .3268\ .3871\ .4342\ .4666\ .4830]^T$
$\{\phi\}_2 = [.2554\ .4342\ .483\ .3871\ .1752\ -.0891\ -.3268\ -.4666]^T$
$\omega_1^2 = 0.0341 \qquad \omega_2^2 = 0.2996$

The damage Ritz vector or DRV $\{r\}$ is defined following the construction of Chu and Milman [10], as follows: assume that the system stiffness change is due to a reduction in the stiffness of a single "damaged" spring, the location of which is assumed known. Apply equal and opposite unit static loads to the two masses to which the damaged spring is connected. The load vector $\{b\}$ which results is a "damage location vector," because its entries identify the location of the damaged spring. For example, if spring 5, which connects the fourth and fifth masses, is damaged, then $\{b\} = [0\ 0\ 0\ 1\ -1\ 0\ 0\ 0]^T$. The DRV $\{r\}$ is the static displacement pattern resulting from the application of the static load $\{b\}$,

$$\{r\} = [K]^{-1}\{b\} \qquad (4)$$

based on the undamaged stiffness matrix [K]. A useful property of this construction is that the change [ΔK] in the stiffness matrix is given for the system of Figure 1 by [10]

$$[\Delta K] = \alpha\{b\}\{b\}^T \tag{5}$$

where $\alpha$ is a proportionality constant which defines the change in spring stiffness. Equation (5) has three nice features: 1) the correct sparseness in [ΔK] is automatically preserved, 2) if the damage location can be identified, so that {b} is determined, then there is a single unknown proportionality constant to be determined, and 3) use of equation (5) to calculate residuals according to the right hand side of equation (2) will automatically put zeroes where they should be in the residual vectors. Equation (5) is used in a number of ways in the sequel.

The basic idea we use for mode shape expansion is to express the damaged system mode shapes as a linear combination of the original, undamaged system mode shapes, plus a Ritz vector which characterizes the effect of the damage [11]. More than one Ritz vector can be used, and this may be necessary for accurate expansion of higher modes. This construction seems natural, because the damaged system mode shape is likely to be similar to the undamaged mode shape, with the Ritz vector used essentially to characterize the *change* in the mode shape due to the damage.

Our mode shape expansion method is illustrated here for the system of Figure 1. Assume the damage consists of a 25% reduction in the stiffness of spring 5 which connects masses 4 and 5. Then the first two mode shapes and natural frequencies of the damaged system are the following:

$\{\psi\}_1 = [.0856\ .1683\ .2455\ .3147\ .3937\ .4390\ .4704\ .4864]^T$
$\{\psi\}_2 = [.2565\ .4398\ .4976\ .4135\ .1441\ -.0991\ -.3140\ -.4393]^T$
$\omega_1^2 = 0.0329 \qquad \omega_2^2 = 0.2583$

Assume the experimental results consist of measurement of the three coordinates $x_2$, $x_5$ and $x_7$, so that the measured data are the aforementioned natural frequencies and the reduced modal matrix

$$[\tilde{\Psi}] = \begin{bmatrix} .1683 & .4398 \\ .3937 & .1441 \\ .4704 & -.3140 \end{bmatrix}$$

There is assumed to be zero measurement error. The following two steps are executed to do the mode shape expansion: first determine the particular linear combination of the reduced versions of the Ritz vector and the first two undamaged mode shapes, such that the preceding measured, reduced mode shapes are exactly reproduced. That is, put

$$[\tilde{\Psi}] = [\tilde{\theta}][W] \tag{6}$$

where the square matrix $[\tilde{\theta}] = \left[\{\tilde{\phi}\}_1\ \{\tilde{\phi}\}_2\ \{\tilde{r}\}\right]$. Solving equation (6) for the matrix [W],

$$[W] = [\tilde{\theta}]^{-1}[\tilde{\Psi}] \tag{7}$$

This result for [W], if used in equation (6), will exactly reproduce the reduced measured mode shapes of the damaged system. The second step in the mode shape expansion is to apply the [W] given by equation (7) to obtain the expanded mode shapes as the same linear conbination of the first two undamaged mode shapes and the Ritz vector, that is, the mode shape expansion is defined by

$$[\hat{\Psi}] = [\theta][W] = [\theta][\tilde{\theta}]^{-1}[\tilde{\Psi}] \tag{8}$$

In the present example (25% reduction in spring 5 stiffness) the expanded mode shapes calculated from equation (8) turn out to be the following:

$\{\hat{\psi}\}_1 = [.0856\ .1683\ .2456\ .3145\ .3931\ .4388\ .4704\ .4864]^T$
$\{\hat{\psi}\}_2 = [.2561\ .4398\ .5002\ .4232\ .1441\ -.0956\ -.313\ -.4414]^T$

Our procedure is similar to what Levine-West et al do in their mode shape expansion procedure using orthogonal procrustes [8]: they find a transformation based on the reduced, measured modes, then they apply this same transformation to the expanded version of the problem. Comparison with the exact, damaged system mode shapes $\{\psi\}_1$ and $\{\psi\}_2$ shows the mode shape expansion done here to be reasonably accurate. The reason for the accuracy here is that the stiffness reduction in spring 5 causes an increase in the relative displacement between masses 4 and 5 in the lower modes. This increased relative displacement, which occurs only between masses 4 and 5, amounts to a rigid body motion. Thr Ritz vector here is $\{r\} = [0\ 0\ 0\ 0\ 1\ 1\ 1\ 1]^T$, which is a rigid body motion of masses 5 through 8. Thus, the Ritz vector is essentially proportional to the change in the mode shape due to the damage.

## 3. DRV BASED MODEL UPDATING

The mode shape expansion described in Section 2 provides an initial approximation for the expaned mode shapes of the damaged system. The associated change [ΔK] in the stiffness matrix is then determined iteratively using the steps described here, which are applied on a mode by mode basis. The procedure presented here is based on a "splitting" of the residual (equation (2)) into a part due to the change in system stiffness matrix and a part due to error in the mode shape. This latter "mode shape error induced" part of the residual is used to update the expanded mode shape. A key limiting assumption is that the damage location is known (at least approximately), for example, through application of one of the methods described in [6,7]. The steps in our iterative model updating calculation are summarized below.

Starting with the residual in the form $\{R\}_r = [\Delta K]\{\hat{\psi}\}_r$, use equation (5) for $[\Delta K]$, so that the rth mode residual is written as

$$\{R\}_r = \alpha\{b\}\{b\}^T\{\hat{\psi}\}_r \tag{9}$$

where $\{b\}$ is specified from the known damage location. Premultiply equation (9) by $\{\hat{\psi}\}_r^T$ and solve the resulting scalar equation to obtain the estimate $\hat{\alpha}$,

$$\hat{\alpha} = \frac{\{\hat{\psi}\}_r^T\{R\}_r}{\{\hat{\psi}\}_r^T\{b\}\{b\}^T\{\hat{\psi}\}_r} \tag{10}$$

where in equation (10) $\{R\}_r$ is calculated via the left hand part of equation (2). Use the estimate $\hat{\alpha}$ to estimate the change in the stiffness matrix as

$$\left[\Delta\hat{K}\right] = \hat{\alpha}\{b\}\{b\}^T \tag{11}$$

Obtain an updated calculation of the residual as

$$\left\{\hat{R}\right\}_r = \left[\Delta\hat{K}\right]\{\hat{\psi}\}_r \tag{12}$$

Equation (12) is considered to be an estimate of the exact residual, that is, the part of the residual due solely to the change in the stiffness matrix. Equation (12) is used to obtain an estimate of that part $\{\Delta R\}_r$ of the residual which is due to the error in the expanded mode shape $\{\hat{\psi}\}_r$,

$$\{\Delta R\}_r = \left\{\hat{R}\right\}_r - \left([K] - \omega_{er}^2[M]\right)\{\hat{\psi}\}_r \tag{13}$$

The expanded mode shape error induced residual $\{\Delta R\}_r$ is then used to obtain an incremental correction $\{\Delta\psi\}_r$ to the expanded mode shape as

$$\{\Delta\psi\}_r = \left([K] - \omega_{er}^2[M]\right)^{-1}\{\Delta R\}_r \tag{14}$$

The updated expanded mode shape is obtained as the normalized version of $\{\psi\}_r + \{\Delta\psi\}_r$. At this stage one iteration is complete and the calculation sequence of equation (10) through equation (14) is repeated until suitable convergence is achieved.

The sequence of calculations is summarized in Appendix 1 for damage consisting of a 10% reduction in the stiffness of spring 3 (corresponding to an exact value $\alpha = 0.10$). The updating procedure using measured mode 1 produces an initial estimate of $\hat{\alpha} = 0.1006$, which is essentially exact to begin with, due to the accuracy of the first expanded mode shape. Appendix 1 shows results for the mode 2 calculation. The first two columns are the exact mode shape and the approximation obtained using the DRV mode shape expansion. The third column is the residual calculated from equation (2). This is used in equation (10) to obtain the first approximation $\hat{\alpha} = 0.1261$. Then equation (12) yields the estimate of the exact residual shown in column 4 of Table 1. Column 5 contains the estimate $\{\Delta R\}_2$ given by equation (13). Column 6 contains $\{\Delta\psi\}_2$ from equation (14), and the resulting updated expanded mode shape, essentially exact, is in column 7. The updated $\hat{\alpha}$ which results is then found to be $\hat{\alpha} = 0.1015$, so the convergence is rapid in this example.

## 4. MODEL REDUCTION BASED UPDATING

In this section we propose a second updating scheme which utilizes a reduced dynamic model having the same number of degrees of freedom as the number m of measured modes. In this scheme m of the n measured coordinates are selected as master degrees of freedom, in terms of which the reduced model is formulated. A procedure is presented for finding the change $[\Delta K]$ in the system stiffness matrix by doing iterative calculations using the reduced model. One advantage of this type of procedure is that mode shape expansion is obviated, so that errors induced by inexact mode shape expansion are not involved.

### 4.1 Model Reduction Technique

The model reduction method we use is described by Burton and Young [12]. This method is similar in spirit to the Guyan reduction [13,14]. Unlike the Guyan reduction, however, our method preserves eigenstructure exactly, that is, the m natural frequencies and mode shapes calculated for the reduced model are the same as the lowest m modes of the full, N degree of freedom model. Here the relevant results of the model reduction, described in [12, 15], are stated. Assume, as in the example of Figure 1, that the mass matrix is the identity matrix. Partition the coordinate vector $\{x\}$ into the so called master and slave coordinate vectors $\{x\}_m$ and $\{x\}_s$, where the m coordinates in $\{x\}_m$ are chosen from among the n measured coordinates. Then partition the stiffness terms in equation (1) as follows:

$$[K]\{x\} = \begin{bmatrix} [k_{ss}] & [k_{sm}] \\ [k_{ms}] & [k_{mm}] \end{bmatrix}\begin{Bmatrix} \{x\}_s \\ \{x\}_m \end{Bmatrix}$$

The reduced model of the system is then of the form

$$[\tilde{m}]\{\ddot{x}\}_m + \left[\tilde{k}\right]\{x\}_m = \{0\} \tag{15}$$

where $[\tilde{m}]$ and $[\tilde{k}]$ are the mxm reduced system mass and stiffness matrices. In order to define the reduced model of equation (15), one needs a slave/master coordinate transformation matrix [T], defined so that

$$\{x\}_s = [T]\{x\}_m \tag{16}$$

If the sxm matrix [T] is specified, then the reduced system matrices are given by

$$[\tilde{m}] = [R]^T[M][R] \tag{17}$$

$$[\tilde{k}] = [R]^T[K][R] \tag{18}$$

where the Nxm "reduction matrix" [R] is defined as

$$[R] = \begin{bmatrix} [T] \\ [I] \end{bmatrix} \tag{19}$$

The crux of any model reduction method based on equation (16) for elimination of unwanted degrees of freedom is the specification of an appropriate transformation matrix [T]. In [12,15] it is shown that the model reduction preserves exactly the lowest m mode eigenstructure if [T] is determined as the solution to the implicit nonlinear matrix-algebraic equation

$$[T] = -\left([k_{ss}] - [T][k_{ms}]\right)^{-1}\left([k_{sm}] - [T][k_{mm}]\right) \tag{20}$$

which applies if [M] is the identity matrix; the more general version of equation (20) for arbitrary [M] is discussed in [12,15]. Note that if [T] is set to zero on the right hand side of equation (20), the relation reduces to the Guyan transformation [13,14]. It turns out that, in terms of the m lowest complete mode shapes of the system being reduced, the transformation matrix can also be expressed as

$$[T] = [\phi]_s[\phi]_m^{-1} \tag{21}$$

where $[\phi]_m$ is the mxm matrix of master degree of freedom eigenvectors for the lowest m modes, and where $[\phi]_s$ is the sxm matrix of slave coordinates for the same modes. The model reduction method can be applied to both the undamaged and the damaged systems. The damaged system transformation matrix [T] will differ from the undamaged system transformation matrix, as will the reduced mass and stiffness matrices for the two systems. For the damaged system the transformation matrix [T] is unknown *apriori*, as are the damaged system reduced mass and stiffness matrices (note that the reduced mass matrix does not retain the diagonal, identity form of the full system mass matrix).

**4.2 Updating Via the Reduced Model**

A model updating method which uses the reduced dynamic model is described here. We first note that the damaged system eigenproblem may be stated for the rth mode as

$$\left([K] - [\Delta K] - \omega_{er}^{2}[M]\right)\{\psi\}_r = \{0\} \tag{22}$$

The *reduced* model eigenproblem for the damaged system may be stated in the form

$$\begin{aligned}(&[R]^T[K][R] - \omega_{er}^{2}[R]^T[M][R] \\ &\quad - \alpha[R]^T[B][R])\{\tilde{\psi}\}_r = \{0\}\end{aligned} \tag{23}$$

where equation (5) has been used for [ΔK], $[B] = \{b\}\{b\}^T$, and the reduced system mass and stiffness matrices have been written out as defined in equations (17) and (18). Assuming the damage location to be known, the unknowns in equation (23) are the damaged system reduction matrix [R] (due to the damaged system transformation matrix [T] being unknown) and the scalar $\alpha$, with $\omega_{er}$ and $\{\tilde{\psi}\}_r$ known from measurement.

In the model updating procedure described here, it is necessary to determine iteratively the scalar $\alpha$ (which specifies [ΔK]) and the damaged system transformation matrix [T]. The following steps are executed in the iteration:

1) Specify an initial approximation for the transformation matrix [T]. Obvious choices include the undamaged system transformation matrix or the transformation matrix associated with the DRV based mode shape expansions of the m lowest modes. Either of these initial approximations can be found from equation (21).

2) Use equation (23) to obtain an estimate $\hat{\alpha}$ by determining the value $\hat{\alpha}$ which renders zero the determinant of the coefficient matrix in equation (23). This result provides an estimate of the change [ΔK] in the system stiffness matrix.

3) Use the estimated [ΔK] to update the four system submatrices $[k_{ss}]$, $[k_{sm}]$, $[k_{ms}]$, and $[k_{mm}]$.

4) Use these updated submatrices to obtain an updated approximation to the damaged system transformation matrix [T]. This is done by solving equation (20) via Gauss-Seidel iteration, starting with the initial approximation for [T], along with the updated submatrices, in the right hand side of equation (20) to produce an updated approximation for [T], which is then used on the right hand side of equation (20) to obtain another updated approximation, etc., until convergence is achieved. The resulting solution for [T] is the damaged system transformation matrix (and the reduction matrix [R]) which is consistent with the approximation [ΔK] from step 2.

5) The updated reduction matrix [R] is inserted into equation (23) and the steps of the iteration are repeated until suitable convergence is achieved.

The reduced model based updating procedure is illustrated for the system of Figure 1 for the following case: the damage consists of a 20% reduction in the stiffness of spring 7, which connects masses 6 and 7. There are assumed to be three measured coordinates: $x_6$, $x_7$, and $x_8$, which are the master degrees of freedom in the three degree of freedom reduced model to be used to do the model updating. This is actually a poor choice of master degrees of freedom, which would normally be more evenly spaced through the structure. The numerical results are shown in Appendix 2 for a calculation based on the first measured mode. In Appendix 2 the matrices $[\phi]_m$ and $[\phi]_s$ for the undamaged system, $[\psi]_m$ and $[\psi]_s$ for the damaged system, and the transformation matrices [T] for the undamaged and damaged systems are first shown, for reference. Use of the *undamaged* transformation/reduction matrices in equation (23) yields an estimate $\hat{\alpha} = 0.225$ needed to render zero the determinant of the coefficient matrix in equation (23). This estimate for $\hat{\alpha}$ leads to an estimate for $\left[\Delta\hat{K}\right]$ according to equation (11). This estimated stiffness matrix change is then used to update the stiffness sub-matrices in equation (20), which is solved iteratively to yield the updated transfornation matrix shown in Appendix 2 (result of five iterations). This updated [T], when inserted into equation (23), yields an updated value $\hat{\alpha} = 0.2003$, which is essentially exact.

## 5. CONCLUDING REMARKS

Three things have been presented: 1) a damage Ritz vector based mode shape expansion method, 2) a companion model updating method in which the stiffness matrix change $[\Delta K]$ and the expanded mode shapes are iteratively refined, and 3) an iterative updating method based on use of the reduced model of the damaged system. A limitation of the work is that the damage location is assumed to be known. Further work is needed to apply the methods to more realistic systems, to study the accuracy and convergence properties, to develop error estimates, and to identify the damage location as part of the procedure. The results presented indicate that the methods presented are promising.

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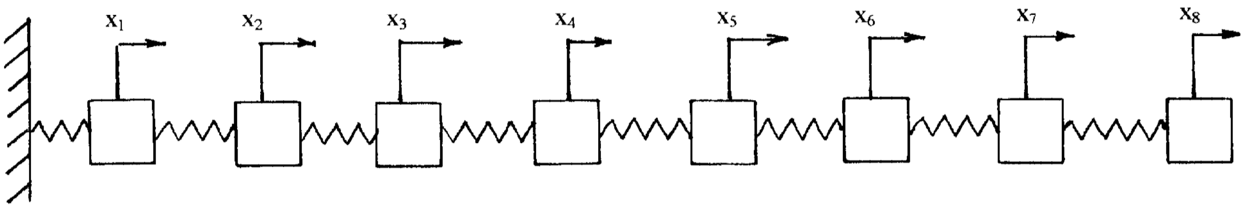
**Figure 1:** The eight degree of freedom system. All masses and stiffnesses of the undamaged system are unity.




**Appendix 1**: Numerical results for the updating example of Section 3 of text; mode 2 calculation.

| $\{\psi\}_{2_{exact}}$ | $\{\psi\}_{2_{DRV}}$ | $\{R\}_2$ | $\{\hat{R}\}_{2_{ex}}$ | $\{\Delta\hat{R}\}_2$ | $\{\Delta\psi\}_2$ | $\{\psi\}_{2_{update}}$ |
|---|---|---|---|---|---|---|
| .2531 | .2535 | .0005 | 0 | -.0005 | .0062 | .2531 |
| .4305 | .4305 | -.0069 | -.0056 | .0013 | .0111 | .4305 |
| .4844 | .4856 | .0070 | .0056 | -.0014 | .0114 | .4844 |
| .3880 | .3883 | -.0005 | 0 | .0005 | .0097 | .3880 |
| .1754 | .1754 | -.0003 | 0 | .0003 | .0045 | .1754 |
| -.0897 | -.0898 | -.0001 | 0 | .0001 | -.0022 | -.0897 |
| -.3279 | -.3280 | .0000 | 0 | .0000 | -.0085 | -.3229 |
| -.4680 | -.4680 | .0000 | 0 | .0000 | -.0122 | -.4680 |

**Appendix 2**: Numerical results for the updating example of Section 4 of text.

$$[\phi]_m = \begin{bmatrix} .4342 & .0891 & -.3268 \\ .4666 & .3268 & .0891 \\ .4830 & .4666 & .4342 \end{bmatrix} \quad [\phi]_s = \begin{bmatrix} .0891 & -.2554 & .3871 \\ .1752 & -.4342 & .4666 \\ .2554 & -.4830 & .1752 \\ .3268 & -.3871 & -.2554 \\ .3871 & -.1752 & -.4830 \end{bmatrix} \quad [T] = \underbrace{\begin{bmatrix} 7.453 & -17.11 & 10.01 \\ 11.74 & -26.42 & 15.33 \\ 11.58 & -24.96 & 14.24 \\ 8.107 & -15.94 & 8.783 \\ 3.874 & -5.893 & 3.012 \end{bmatrix}}_{\text{undamaged system}}$$

$$[\psi]_m = \underbrace{\begin{bmatrix} .4308 & .0492 & -.3776 \\ .4713 & .3429 & .0995 \\ .4878 & .4800 & .4061 \end{bmatrix}}_{\text{measured data}} \quad [\psi]_s = \begin{bmatrix} .0883 & -.2434 & .3802 \\ .1736 & -.4172 & .4733 \\ .2530 & -.4719 & .2092 \\ .3239 & -.3919 & -.2129 \\ .3839 & -.1999 & -.4743 \end{bmatrix} \quad [T] = \underbrace{\begin{bmatrix} 6.447 & -17.25 & 11.16 \\ 10.21 & -26.80 & 17.23 \\ 10.16 & -25.55 & 16.23 \\ 7.218 & -16.50 & 10.23 \\ 3.528 & -6.159 & 3.622 \end{bmatrix}}_{\text{damaged system}}$$

$$[T] = \underbrace{\begin{bmatrix} 6.355 & -17.42 & 11.42 \\ 10.07 & -27.06 & 17.64 \\ 10.02 & -25.80 & 16.62 \\ 7.119 & -16.66 & 10.49 \\ 3.486 & -6.219 & 3.724 \end{bmatrix}}_{\text{damaged system, updated}}$$